

Flux tube delocalization at the deconfinement point.

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Abstract

We study the behaviour of the flux tube thickness in the vicinity of the deconfinement transition. We show, using effective string methods, that in this regime the square width increases linearly and not logarithmically with the interquark distance. The amplitude of this linear growth is an increasing function of the temperature and diverges as the deconfinement transition is approached from below. These predictions are in good agreement with a set of simulations performed in the 3d gauge Ising model.

1 Introduction

One of the most intriguing features of the confining regime of Lattice Gauge Theories (LGTs) is the logarithmic increase of the square width $w^2(R)$ of the flux tube as a function of the interquark distance R [1]. This effect, which is commonly referred to as the "delocalization" of the flux tube is together with the well known Lüscher term one of the most important and stringent predictions of the effective string approach to LGTs.

This effect was discussed for the first time many years ago by Lüscher, Münster and Weisz in [1] but it required several years of efforts before it could be observed in lattice simulations. The first numerical results were obtained in abelian models [2–5] and only recently they were extended also to non abelian LGTs [6,7] (see also [8] for some early attempt). In all these tests a good agreement with the theoretical predictions was found.

A question which naturally arises in this context is which is the fate of the flux tube (and in particular the behaviour of its width) as the deconfinement transition is approached from below. It is important to stress that delocalization and deconfinement are two deeply different conditions of the flux tube. The deconfinement transition is characterized by the vanishing of the string tension $\sigma(T)$. In pure gauge theories it is associated to the breaking of the center symmetry of the gauge group and its natural order parameter is the Polyakov loop. At the deconfinement point the flux tube vanishes. The delocalization of the flux tube instead coincides with the roughening transition. In all the physically interesting models this transition occurs for value of the gauge coupling β well below the fixed point where the continuum limit can be taken. Thus in the continuum limit (for any temperature below the deconfinement transition) the theory is always in the rough phase and thus the flux tube is always delocalized. Delocalization is a typical quantum effect. It is a consequence of the Mermin-Wagner theorem which imposes the restoring (in the continuum limit) of the translational symmetry for the fluctuations of the flux tube in the transverse directions. Intuitively it amounts to say that we cannot fix deterministically the trajectory of the flux tube but may only describe it as a probability distribution. It is important to stress that, even if delocalized, the flux tube fully keeps its confining function. The quantum fluctuations which drive the delocalization also influence the confining potential (as the presence of Lüscher term indicates) but do not destroy it.

While the behaviour of the string tension $\sigma(T)$ as the deconfinement temperature T_c is approached from below is rather well understood much less is known on the behaviour of the flux tube thickness in this regime.

This is a rather important issue from a physical point of view since the interplay between delocalization and deconfinement could strongly influence the transition from hadrons to free quarks as T_c is approached.

This problem can be addressed within the framework of the effective string approach by performing a modular transformation of the low temperature result. This was done in [9] in the case of the free bosonic approximation (i.e. the first order in the perturbative expansion of the Nambu-Goto effective string). In this free bosonic limit one can show that when periodic boundary conditions are imposed in the time direction (i.e. when finite temperature regularization is imposed) the large R behaviour of the square width changes completely and becomes linear instead of logarithmic. This behaviour holds in principle for any temperature T , but as T decreases it requires larger and larger values of R to be observed. Similarly it is possible to show that for any fixed value of R the square width smoothly converges toward the expected logarithmic behaviour as T decreases. The threshold between the two behaviours is $R \sim 1/T$. The coefficient of the linear dependence at high T is itself temperature dependent. It increases linearly with the temperature and remains finite as the deconfinement transition is approached.

In [9] these predictions were compared with a set of high precision simulations in the 3d gauge Ising model, with results which turned out to be only in partial agreement with the effective string picture. For all the temperatures studied in [9] $w^2(R)$ was indeed, (with very good confidence levels) a linearly increasing function of R . However the coefficient of this linear behaviour was in general larger than that predicted by the effective string (except for the smallest temperature values) and, what is more important, it seemed to diverge as the deconfinement point was approached.

It is clear that in order to understand this discrepancy one should go beyond the first order approximation in the effective string calculation. As a matter of fact in this finite temperature description, the perturbative expansion in powers of $1/\sigma_0 L^2$, is actually an expansion in T^2/T_c^2 (see below for further details on this point) thus it is somehow obvious that higher orders terms become important if one wants to study the behaviour in the vicinity of the deconfinement point. What is less obvious is how relevant are for the flux tube width these higher order contributions. Indeed it seems that the particular problem that we are addressing here, the high temperature behaviour of the flux tube thickness, is one of the best numerical laboratories to test higher order contributions in the effective string description. In fact the comparison with the 3d Ising model [9] shows that these higher order corrections may become very large (much larger than the first order term)

in the vicinity of the deconfinement transition.

Higher order effective string corrections have attracted much interest in these last years both as a tool to better fit the numerical results and as a way to improve our understanding of the interplay between the effective string approach and standard string theory. However no result had been obtained until very recently on higher order corrections to the flux tube width. Most of the papers concentrated instead on higher order corrections to the interquark potential. Several important results were obtained in this context. It was shown that not only the leading (Lüscher) term but also the first subleading (quartic) correction is universal [10–12], thus remarkably enhancing the predicting power of the effective model. In a recent paper universality has been extended also to the subsequent term (sixth order term in the potential) [13] at least for the (2+1) dimensional case and for the torus and cylinder topologies. Finally, in the case of the Nambu-Goto string, the all order expansion was obtained in [10, 14].

The situation changed recently thanks to [6] where an explicit expression for the next to leading (quartic) correction to the flux tube width was obtained in the case of the cylinder topology. This is exactly the topology which is needed to describe the behaviour of the flux tube at finite temperature.

The aim of the present paper is to use the result of [6] to address the discrepancy between theoretical predictions and numerical results described above. As we shall see, keeping into account this correction the right T dependence of the flux tube thickness is recovered. This, together with a parallel result obtained using a dimensionally reduced effective model for the high temperature behaviour of lattice gauge theories, will allow us to guess further terms in the expansion and to propose the following conjecture for the behaviour for the large R limit of the flux tube square width up to the deconfinement transition:

$$\sigma(T)w^2(R) = \frac{1}{4}RT \quad (1)$$

where $\sigma(T)$ denotes the temperature dependent string tension.

If, in addition, one also assumes a Nambu-Goto effective action for the flux tube (assumption which we know to be wrong, but in several cases gives a very good approximation, at least for the first few orders in T) then the T dependence of $\sigma(T)$ can be predicted exactly and turns out to be

$$\sigma(T) = \sigma_0 \left(1 - \frac{T^2}{T_c^2}\right)^{1/2}. \quad (2)$$

Thus allowing a complete description of the T dependence of the flux tube thickness in the limit of large interquark separation.

2 Effective string prediction for the flux tube thickness at finite temperature

2.1 Definition of the flux tube thickness

In a finite temperature setting the lattice operator which is used to evaluate the flux through a plaquette p of the lattice is:

$$\langle \phi(p; P, P') \rangle = \frac{\langle PP'^{\dagger} U_p \rangle}{\langle PP'^{\dagger} \rangle} - \langle U_p \rangle \quad (3)$$

where P, P' are two Polyakov loops separated by R lattice spacings and U_p is the operator associated with the plaquette p . Different possible orientations of the plaquette p measure different components of the flux. In the following we shall neglect this dependence which plays no role in our analysis. The only information that we need is the position of the plaquette. Let us define

$$\langle \phi(p; P, P') \rangle = \langle \phi(\vec{h}; R, L) \rangle$$

where \vec{h} denotes the displacement of p from the $P P'$ plane. In each transverse direction, the flux density shows a gaussian like shape (see for instance Fig. 2 of [2]). The width of this gaussian w is the quantity which is usually denoted as “flux tube thickness”:

$$w^2(R, L) = \frac{\sum_{\vec{h}} \vec{h}^2 \langle \phi(\vec{h}; R, L) \rangle}{\sum_{\vec{h}} \langle \phi(\vec{h}; R, L) \rangle} \quad (4)$$

This quantity depends on the number of transverse dimensions and on the bare gauge coupling β (or, equivalently, on the lattice spacing a). Once β is fixed the only remaining dependences are on the interquark distance R and on the lattice size in the compactified timelike direction L , i.e. on the inverse temperature of the model. By tuning L we can thus study the flux tube thickness near the deconfinement transition.

2.2 Effective string prediction: general setting

In the effective string framework the square of flux tube width is given by:

$$w^2(x; R, L) = \frac{\int_{\mathcal{C}} [\mathcal{D}h] h^i(t, x) h^i(t, x) e^{-S[h]}}{\int_{\mathcal{C}} [\mathcal{D}h] e^{-S[h]}} \quad (5)$$

where the sum is intended over all the surfaces bordered by the two Polyakov loops and $S[h]$ is the effective action. As mentioned above, up to the second order in which we are interested here, this action is universal and coincides with the expansion to the fourth order in h of the Nambu-Goto action. In the following we shall concentrate in the (2+1) dimensional case (i.e. we shall deal with only one transverse direction) and eliminate the index of the field h . In this case the Nambu Goto action has a particularly simple form

$$S[h] = \sigma_0 \int_{-L/2}^{L/2} d\tau \int_{-R/2}^{R/2} d\zeta \sqrt{1 + (\partial_\tau h)^2 + (\partial_\zeta h)^2} . \quad (6)$$

The correlator eq.(5) is singular and must be regularized. The standard choice is the point splitting regularization. It is easy to see looking at eq.(6) that, in the large R/L limit in which we are interested there is a natural expansion parameter in the evaluation of eq.(5) which is $1/(\sigma_0 L^2)$. The leading order in this expansion is simply given by the free bosonic approximation to the Nambu-Goto action while the next to leading correction will include the fourth order terms of the action. Let us address these two contributions in more detail.

2.3 Effective string prediction: leading order

The leading order effective string prediction for square of the flux tube thickness w_{lo}^2 in the finite temperature (i.e. cylinder) geometry was evaluated for the first time (in a rather implicit form) in [2]. It was then reobtained in a slightly more explicit form in [9] using the method of images. We shall use as our starting point this last expression:

$$\sigma_0 w_{lo}^2 = -\frac{1}{2\pi} \log \frac{\pi|\epsilon|}{2R} + \frac{1}{2\pi} \log |\theta_2(0)/\theta_1'(0)| \quad (7)$$

where σ_0 denotes the zero temperature string tension and L is the length of the cylinder in the compactified direction (i.e. the inverse temperature) and

the square width is evaluated at the midpoint between the two Polyakov loops.

It is easy to see using standard relations among θ and η functions that this expression coincides with the one reported in [6].

In the large R and high temperature limit, i.e. for $R \gg L$ this expression becomes:

$$\sigma_0 w_{lo}^2 = \frac{1}{2\pi} \log \frac{L}{L_c} + \frac{R}{4L} - \frac{1}{\pi} e^{-2\pi \frac{R}{L}} + \dots \quad (8)$$

which shows, as anticipated, that the square width should increase linearly with R with a coefficient $\frac{1}{4\sigma_0 L}$ which increases linearly with the temperature and stays finite at the deconfinement point. In fact the zero temperature string tension σ_0 (which can be extracted from the Wilson loop or from Polyakov loop correlators at very low temperature) does not depend on the finite temperature T but only on the coupling constant β .

2.4 Effective string prediction: next to leading correction

The next to leading correction was recently evaluated in [6]. We report here for completeness the result:

$$\begin{aligned} w_{nlo}^2 &= \left(1 + \frac{4\pi f(\tau)}{\sigma_0 r^2}\right) w_{lo}^2(r/2) - \frac{f(\tau) + g(\tau)}{\sigma_0^2 r^2}, \\ f(\tau) &= \frac{E_2(\tau) - 4E_2(2\tau)}{48}, \\ g(\tau) &= i\pi\tau \left(\frac{E_2(\tau)}{12} - \frac{qd}{dq}\right) \left(f(\tau) + \frac{E_2(\tau)}{16}\right) + \frac{E_2(\tau)}{96}, \end{aligned} \quad (9)$$

where $E_2(\tau)$ is the first Eisenstein function:

$$E_2(\tau) = 1 - 24 \sum_{n=1}^{\infty} \frac{n q^n}{1 - q^n}.$$

$\tau \equiv \frac{iL}{2R}$ denotes the modulus of the cylinder and

$$q = e^{2\pi i\tau} = e^{-\pi L/R}$$

The dominant term in the $R \gg L$ limit (i.e. $\tau \rightarrow 0$) turns out to be again linear in R :

$$w_{nlo}^2 \sim \frac{\pi R}{24\sigma_0^2 L^3} + \dots \quad (10)$$

This result is rather non trivial. Looking at the small τ expansion of the Eisenstein function one would in principle expect a term proportional to R^2 which however disappears in eq.(9) as a consequence of a set of non trivial cancellations among Eisenstein functions. This is an important consistency check in view of the fact that the simulations reported in [9] found no evidence of terms proportional to R^2 and found instead a remarkably precise linear increase of w^2 with R .

Combining together the leading and subleading corrections we end up with the following expression:

$$w^2 = \frac{R}{4\sigma_0 L} \left(1 + \frac{\pi}{6\sigma_0 L^2} \right) + \dots \quad (11)$$

which, assuming the Nambu-Goto value for the critical temperature $T_c^2 = 3\sigma_0/\pi$ (see below for a derivation of this result), can be rewritten as an expansion in T/T_c

$$w^2 = R \frac{3}{4\pi} \frac{T}{T_c^2} \left(1 + \frac{T^2}{2T_c^2} \right) + \dots \quad (12)$$

We compare in tab.1 and fig.1 (dashed line) this result with the values of the numerical simulations reported in [9]. We see a clear improvement with respect to the leading contribution alone (continuous line), which however is not enough to fill the gap with the numerical data. At the same time it is easy to see that, as anticipated, for values of L in the vicinity of the deconfinement point the subleading term becomes of the same order of the leading one. This shows that in this limit the expansion in powers of $1/\sigma_0 L^2$ converges too slowly and that it would be very important to have some educated guess on how to resum the expansion. We shall show in the next section that this guess can be obtained with a completely different approach, i.e. looking at the effective theory obtained integrating out the spacelike degrees of freedom following the well known Svetitsky-Yaffe proposal [15].

2.5 Dimensional reduction and the Svetitsky Yaffe approach.

In the vicinity of the deconfinement transition the physics of a $(d+1)$ LGT can be described using an effective model in which the spacelike links are integrated out and the only remaining degrees of freedom are the Polyakov loops. This essentially amounts to integrate out all the degrees of freedom

of the original LGT up to the scale $L = 1/T$ and in fact the theory obtained in this way effectively behaves as a model in one dimension less than the original LGT. It is easy to see that this procedure gives as effective description a d dimensional spin model (the Polyakov loops playing the role of spins) with a global symmetry represented by the center C of the gauge group of the original LGT. This approach was pioneered by Svetitsky and Yaffe [15] who were able to predict in this way the critical behaviour of the LGT at the deconfinement point under the assumption that both the deconfinement transition of the original LGT and the magnetization transition of the effective spin model are continuous. However its utility goes beyond the prediction of the critical indices. In the last few years it has been shown that this effective description can be used to predict the behaviour of various quantities in the vicinity of the deconfinement transition as far as the scales involved in these observables (for instance the interquark distance R) are larger than $L = 1/T$ which sets the scale of the effective theory (for a review of most recent results see for instance [16]). This is exactly the $R \gg L$ limit in which we are interested in this paper.

The simplest examples of this effective mapping are the (2+1) $SU(2)$ LGT and the (2+1) Ising gauge model which have the same center Z_2 and are thus both mapped into the 2d spin Ising model. In this case, since both LGTs have a second order deconfinement phase transition we can also predict, following [15], that these transitions are in the same universality class of the 2d Ising model.

Let us address in more detail these examples. It is easy to see that the confining phase of the LGT is mapped into the high temperature phase of the spin model and that, as we mentioned before, the Polyakov loops of the LGT are mapped into the spins of the 2d Ising model. This correspondence should be understood in the renormalization group sense i.e. the Polyakov loop operator is actually mapped into a linear combination of all the (C -odd) operators of the spin model. In the 2d Ising case this means that we have a combination involving the whole conformal family of the spin operator. For R large enough this combination is dominated by the relevant operators which in the Ising case is only the spin operator. In a similar way it is possible to show that the plaquette operator is mapped into the most general combination of the energy and the identity conformal families. This allows us to construct the analogous of the operator which measures the flux tube thickness which turns out to be a suitable combination of three point correlators of the spin and energy operators (see [17] for a detailed discussion of this mapping). In the particular case of the 2d Ising model these correlators can be evaluated exactly leading to the following expression

for the "flux" distribution [17]

$$P(R, y) = \frac{2\pi R}{4y^2 + R^2} \frac{e^{-m\sqrt{4y^2 + R^2}}}{K_0(mR)}. \quad (13)$$

where y denotes the transverse direction, K_0 is the modified Bessel function of order 0, m is the mass of the 2d Ising model and a large mR limit is assumed.

From this flux distribution it is easy to extract the square of the flux tube width as the ratio

$$w^2(R) = \frac{\int_{-\infty}^{\infty} dy y^2 P(R, y)}{\int_{-\infty}^{\infty} dy P(R, y)} \quad (14)$$

which, setting $x = 2y/R$ amounts to evaluate

$$w^2(R) = \frac{R^2}{4} \frac{\int_{-\infty}^{\infty} dx \frac{x^2}{1+x^2} e^{-2mr\sqrt{1+x^2}}}{\int_{-\infty}^{\infty} dx \frac{e^{-2mr\sqrt{1+x^2}}}{1+x^2}} \quad (15)$$

These integrals can be evaluated asymptotically in the large mR limit (see [17]¹) leading to the following result:

$$w^2(R) \simeq \frac{1}{4} \frac{R}{m} + \dots \quad (16)$$

where the dots stay for terms constant or proportional to negative powers of R .

The last step in order to compare this result with eq.(11) is to give a meaning to the Ising mass m in terms of LGT quantities.

This can be easily accomplished if we recall that the mass is the inverse of the correlation length of the model and can be obtained from the large R limit of the 2 point function of the model as follows:

$$\lim_{R \rightarrow \infty} \langle \sigma(0, 0) \sigma(0, R) \rangle \sim K_0(mR). \quad (17)$$

According to the mapping discussed above this correlator is the 2d limit of the expectation value of two Polyakov loops at distance R . In order to be consistent with the assumptions we made to evaluate the flux tube width in the previous section, we must evaluate this correlator in the framework of

¹Notice, to avoid confusion, that in [17] we used the variable $r \equiv R/2$ and that we evaluated the unnormalized square width, i.e. only the numerator of eq.(14)

the Nambu-Goto effective action. This result was obtained a few years ago by Lüscher and Weisz [10] using a duality transformation and then derived in the covariant formalism in [14]. In $d = 2 + 1$ dimensions one finds a tower of K_0 Bessel functions:

$$\langle P(0,0)P(0,R) \rangle = \sum_{n=0}^{\infty} c_n K_0(E_n R). \quad (18)$$

(see eq. (3.2) of [10]).

where c_n are constants (whose exact expression is irrelevant for our analysis, the only important point is that they do not contain any R dependence in $(2+1)$ dimensions, contrary to the higher dimensional cases) and E_n are the closed string energy levels:

$$E_n = \sigma_0 L \left\{ 1 + \frac{8\pi}{\sigma_0 L^2} \left[-\frac{1}{24} (d-2) + n \right] \right\}^{1/2}. \quad (19)$$

(see eq. (C5) of [10]).

It is easy to see that in the large R limit only the lowest state ($n = 0$) survives and, in full agreement with our dimensional reduction mapping, we end up with a single K_0 function:

$$\lim_{R \rightarrow \infty} \langle P(0,0)P(0,R) \rangle \sim K_0(E_0 R). \quad (20)$$

comparing with eq.(17) we immediately recognize $m = E_0$ i.e.

$$m = \sigma_0 L \left(1 - \frac{\pi}{3\sigma_0 L^2} \right)^{1/2}. \quad (21)$$

This result can be rewritten as

$$m = \sigma(T) L \quad (22)$$

where $\sigma(T)$ is given by eq.(2) which we report here:

$$\sigma(T) = \sigma_0 \left(1 - \frac{T^2}{T_c^2} \right)^{1/2}. \quad (23)$$

where the critical temperature $T_c^2 = 3\sigma_0/\pi$ is the Nambu-Goto prediction for the deconfinement temperature and is given by the solution of the equation $\sigma(T) = 0$. Eq.(23) is the prediction of the Nambu-Goto effective action for the temperature dependence of the string tension and was discussed several years ago by Olesen in [18].

Plugging this result in eq.(16) we find for the coefficient of the linear term:

$$w^2(R) = \frac{1}{4} \frac{R}{\sigma(T)L} + \dots = \frac{1}{4} \frac{R}{\sigma_0 L \left(1 - \frac{\pi}{3\sigma_0 L^2}\right)^{1/2}} + \dots \quad (24)$$

which may be expanded in powers of $1/\sigma_0 L^2$ as follows

$$w^2(R) = \frac{1}{4} \frac{R}{\sigma_0 L} \left(1 + \frac{\pi}{6\sigma_0 L^2} + \frac{\pi^2}{24\sigma_0^2 L^4} + \dots\right) \quad (25)$$

or equivalently

$$w^2 = R \frac{3}{4\pi} \frac{T}{T_c^2} \left(1 + \frac{T^2}{2T_c^2} + \frac{3T^4}{8T_c^4}\right) + \dots \quad (26)$$

The first two orders exactly coincide with those of eq.(11). This represents a remarkable consistency check of both the Svetitsky-Yaffe approach and the second order perturbative calculation of [6] and leads to conjecture that eq.(24) could be the resummation to all orders in $1/(\sigma_0 L^2)$ of the linear term of the square width in the framework of the Nambu-Goto effective action.

It is important to stress, as a final comment on this result, that even if it was obtained assuming for $\sigma(T)$ the functional form predicted by the Nambu-Goto action: eq.(2) its validity goes well beyond this particular model. In fact the calculations which lead to eq.(11) and to the first two terms of eq.(26) only involve contributions up to the next to leading order in the effective string perturbative expansion which were proved to be universal in [10–12].

2.6 Comparison with the Ising model.

In fig.1 and tab.1 we compare eq.(24) with the results of a set of simulations performed in the 3d gauge Ising model and reported in [9]. In these simulations the coupling of the Ising model was fixed at $\beta = 0.75180$ for which the deconfinement transition is known to occur at exactly 8 lattice spacings [19]. Moreover for this value of β the zero temperature string tension is known with very high precision to be $\sigma_0 = 0.0105255(11)$ [20] thus excluding a possible source of systematic errors. The model was studied for the values of L reported in the first column of tab.1 which correspond to temperatures

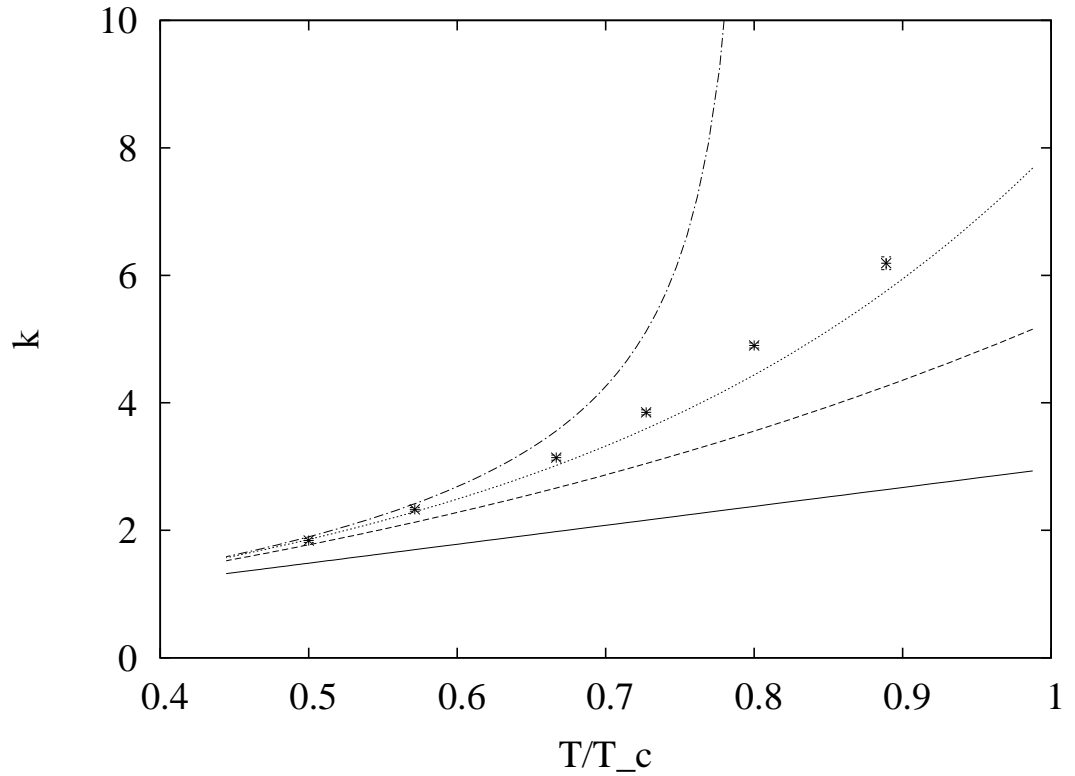


Figure 1: Plot of $k(L)$ as a function of $\frac{T}{T_c} = \frac{8}{L}$. The continuous line is the prediction for $k(L)$ according to eq.(8). The dashed line according to eq.(11). The dashed dotted line to eq.(24) and the dotted line to (25). In all the four cases the curves were obtained only using the terms explicitly written in the corresponding equations and neglecting higher order corrections (denoted with the dots in the various equations). The points are the results of the simulations in the 3d gauge Ising model

L	$k(L)$	eq.(8)	eq.(11)	eq.(24)	eq.(25)
9	6.19(10)	2.639	4.260	NAN	5.753
10	4.90(4)	2.375	3.557	33.307	4.438
11	3.85(4)	2.159	3.047	5.121	3.594
12	3.14(4)	1.979	2.663	3.560	3.017
14	2.33(3)	1.697	2.127	2.418	2.291
16	1.84(3)	1.484	1.773	1.899	1.857

Table 1: Values of the coefficient $k(L)$ of eq.(27) for various values of L . In the second column we list the results of the simulations. In the following columns we report the prediction for $k(L)$ assuming eq.(8)(11) (24)(25) and neglecting higher order corrections denoted by the dots in the equations.

ranging from $T = T_c/2$ to $T = 8T_c/9$. For each temperature w^2 was then evaluated for several values of R and fitted with

$$w^2(R, L) = k(L)R + c(L) \quad (27)$$

For all temperatures the reduced χ^2 of the fits was very good. The values of $k(L)$ obtained in this way are reported in the second column of tab.1. Looking at fig.1 and tab.1 we see that for all the temperatures eq.(24) predicts values of $k(L)$ larger than those found in the simulations and diverges for $T \sim 4T_c/5$. This is not surprising. Indeed it is by now well known that the Nambu-Goto effective string is not a good description of the 3d gauge Ising model. In fact it predicts a deconfinement temperature which is ~ 0.8 of the observed one (in agreement with what we find here) and predicts too large corrections in the interquark potential. This disagreement was discussed in the past years in a set of high precision numerical tests both in the high T [21] (but still in the confining phase) and in the low T [22] regimes of the interquark potential and more recently looking at the interface free energy of the 3d Ising spin model [20]. This last observable is related by duality to the interquark potential and represents an important cross test because it involves different (i.e. toroidal) boundary conditions. In all these tests the interquark potential was rather well described by the truncation at the second order of the Nambu-Goto expansion in powers of $1/\sigma_0 L^2$ and definitely incompatible with any further higher order contribution. Remarkably enough we see the same pattern also in this case. We report in fig.1 (the dotted line) and tab.1 (last column) the values of $k(L)$ obtained truncating eq.(24) at the second order (i.e. eq.(25)). As shown in fig.1 the measured values lie slightly above this curve. This is indeed a puzzling result in view of the recent claim [13] on the universality of the sixth order term in the

effective string expansion for the interquark potential. This sixth order term seems instead to be very different from the Nambu-Goto one (and compatible this zero) in the case of the 3d Ising model. We have for the moment no good answer to this puzzle.

3 Conclusions

Let us summarize our main results and add a few concluding comments:

- 1) As anticipated in the introduction our main result is that approaching the deconfinement transition has a twofold effect on the square width of the flux tube. First, the logarithmic dependence on the interquark distance is changed into a linear one. Second, (see eq.(12)) the amplitude of this linear growth increases more than linearly with the temperature as the transition point is approached.
- 2) We conjecture that the behaviour described by eq.(12) is only the first term of an expansion in powers of T^2/T_c^2 and that the square width w^2 should be proportional to the inverse of the finite temperature string tension (see eq.(24)) If we isolate in eq.(24) the term linear in R we find an universal relation which should hold for any LGT in the limit of large interquark distances

$$\sigma(T)w^2(R) = \frac{1}{4}RT \quad (28)$$

Moreover, if we assume that the effective action is of the Nambu-Goto type, then we may also predict the explicit expression for $\sigma(T)$ which is given by eq.(2).

- 3) However it is well known that for many models (and in particular for the 3d gauge Ising model) the Nambu-Goto picture is not correct and that eq.(2) is a rather poor description of the actual finite temperature behaviour of the string tension $\sigma(T)$. In the particular case of the 3d gauge Ising model we find a good agreement between predictions and numerical data if we truncate at the second order the perturbative expansion in powers of $1/\sigma_0 L^2$ in agreement with what already found when looking at the interquark potential.
- 4) On the contrary in the case of (2+1) $SU(N)$ gauge models (and in particular for $N = 3$) the Nambu-Goto string gives a much better description of the finite temperature behaviour of the theory (see [23])

and accordingly we expect that eq.(24) should well describe the finite temperature behaviour of w^2 . It would be very interesting to extend to these cases the present analysis, using for instance the methods of [6] or of [7] and to test on a quantitative basis our prediction eq.(1).

- 5) For LGTs with a first order deconfinement transition (as it happens for instance for $N \geq 4$ in $d = 2 + 1$ [23] and for $SU(3)$ in $d = 3 + 1$), there is no reason to expect a divergent behaviour of the coefficient $k(L)$. It would be very interesting to see if eq.(1) still holds in these cases.

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